the principal value of the integral is calculated. The integral equation (7) is a boundary impedance constraint on the trial magnetic field in (5) and (6) which precisely represents the effect of the exterior free-space region R_A.

In order to proceed with the method of moments [(5) and (6)], it is necessary to express $\partial \boldsymbol{H}_{tC}/\partial n$ in (7) in terms of $\boldsymbol{H}_{tC}$. This can be achieved by a separate application of the method of moments to (7), which leads to the matrix relation

$$\psi_i = \sum_{j \text{ (over } C)} A_{ij}(k_A{}^2)\phi_j \tag{8}$$

where $\partial \boldsymbol{H}_{tC}/\partial n$ and $\boldsymbol{H}_{tC}$ are represented by the parameters ψ_i and ϕ_j, respectively. The boundary terms in (5) and (6) can therefore be approximated by the linear combinations

$$\oint_C W_i \frac{\partial H_x}{\partial n}\, ds = \sum_{j \text{ (over } C)} B_{ij}(k_A{}^2)\phi_j \tag{9}$$

$$\oint_C W_i \frac{\partial H_y}{\partial n}\, ds = \sum_{j \text{ (over } C)} C_{ij}(k_A{}^2)\phi_j. \tag{10}$$

Hence the application of the method of moments to (1) yields the matrix eigenvalue problem

$$\sum_{j \text{ (over } R \text{ and } C)} D_{ij}(k_A{}^2)\phi_j = \omega^2 \sum_{j \text{ (over } R \text{ and } C)} E_{ij}\phi_j \tag{11}$$

in which the value of $k_A{}^2$ is specified.

Example

The method was applied to the rectangular dielectric rod shown in Fig. 1. This simple example may be analyzed by placing the auxiliary boundary directly on the dielectric–air interface. The approximate field distributions and dispersion characteristics of the first few surface modes were obtained by solving (11) for a range of values of $k_A{}^2$. A coarse square mesh system on a 6 point $\times$ 5 point grid was used which resulted in matrices of order 60. Bilinear expansion functions and testing functions were used for (5) and (6) whereas point matching was used to reduce (7) to a matrix constraint. Fig. 2

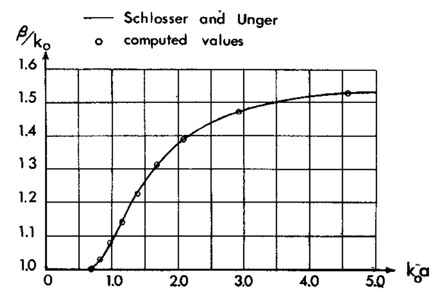

Fig. 2. Dispersion characteristic of the dominant $_0EH_{11}$ surface mode of the dielectric rod ($b/a = 1.25$, $\epsilon/\epsilon_0 = 2.5$).

shows the dispersion characteristic of the dominant $_0EH_{11}$ surface mode for the case $b/a = 1.25$ and relative permittivity 2.5. Good agreement is obtained with the results of Schlosser and Unger [7].

This method is currently being evaluated for obtaining dispersion characteristics of optical fibers and open-boundary structures which can support a quasi-TEM mode.

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Correction for Adapters in Microwave Measurements

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***Abstract*—A measurement on a standard termination provides sufficient information for making corrected measurements through an adapter, if the dissipative loss of the adapter can be neglected. This approximation often gives more consistent results than calibration techniques that require highly reflecting standards.**

Introduction

Since it is impractical to develop measurement equipment for each of the transmission-line and connector types in common use, measurements are often carried out through passive reciprocal adapters (also called "transitions" or "transducers").

Adapters are designed to have low loss and low reflection. However, it is often necessary to apply computed corrections to achieve the desired accuracy. This paper presents a simple method for determining the corrections, based on the assumption that the adapter has negligible dissipative loss. The "primary" connector on the measuring apparatus will usually permit repeatable low-loss connections. The "secondary" connector type on the device under test may or may not permit consistent low-loss connections. If not, the simple method will give more valid results than the technique now commonly used with computer-controlled network analyzers.

Two examples will illustrate the intended applications of the method. In each case, suppose that a computer-controlled network analyzer system is available with 7-mm precision connectors: the "primary" connector system.

The first example is the measurement of devices with SMA connectors. Adapters from 7 mm to the "secondary" SMA connector, constructed with reasonable care, will have good conducting surfaces and negligible dielectric losses. It is probably more accurate to consider such an adapter to be dissipationless than it is to assume that low-loss connection of reference standards can be achieved consistently and repeatedly without excessive stress on the SMA connector.

The second example is the measurement of waveguide components with the 7-mm connector as a "primary" connector. This practice is simply an expedient to avoid setting up measurement apparatus in each of the numerous waveguide bands. The proposed method requires only a precision or sliding load as a reference standard in each waveguide type.

The discussion will consider a single frequency. It will be assumed that the network analyzer is linear or that stored calibrations to correct for nonlinearities have already been applied.

Present Calibration Technique

The present method is based upon carrying out, at the secondary connector, the same sort of calibration procedure as ordinarily used at the primary connector to determine corrections for residuals in the measurement system. For this purpose, reflection standards are required in the secondary connector system. Ordinarily, the standards used are a short circuit, a matched termination (which may be a sliding load), and an open circuit or an offset short circuit. A set of such standards is required for each connector type for which measurements are required.

The reflection coefficient is a bilinear function of the network analyzer output, so the computer program is the same for making corrected measurements at the secondary connector as at the primary connector.

When most of the measurements are to be made on highly reflecting devices, it is preferable to use three highly reflecting standards [1]. The calibration program must be modified in this case.

Simplified Method

In the simplified method, the network analyzer is calibrated for reflection measurements at its primary connector, by reference to reflection standards for the primary connector. Then the adapter is

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attached and terminated with a matched load (which can be simulated by a sliding load) in the secondary connector system. A measurement of this terminated assembly is made and corrected with respect to the primary port and stored as M_L. Reflection measurements on unknown devices in the secondary connector system are next made and initially corrected with respect to the primary connector. Then these corrected measurements are further corrected for the adapter by reference to M_L. The mathematical details will be given below.

The reference plane for reflection phase is arbitrary when the method is used in this fashion. However, phase information is fully utilized in the corrections. Also, phase differences between different unknowns are correctly determined.

A definite reflection phase reference plane can be established by a measurement on another standard, such as a short circuit in the secondary connector system. Losses are not critical when the short circuit is used only for phase reference.

Equations for Correcting Measurements

Let S be the scattering matrix of the adapter, with port 1 the primary connector and port 2 the secondary connector. Then the measurement M at port 1 is given by

$$M = S_{11} + \frac{S_{21}^2\Gamma}{1 - S_{22}\Gamma} \tag{1}$$

where Γ is the reflection coefficient of the device connected to port 2. The adapter is reciprocal, so $S_{12} = S_{21}$ has been assumed. Suppose that the measurement M_L is obtained at port 1 when a perfect termination ($\Gamma = 0$) is attached to port 2. Then, from (1),

$$M_L = S_{11}. \tag{2}$$

From the conjugate matching theorem, maximum power transfer will occur when $\Gamma = S_{22}^*$. For a lossless adapter, maximum power transfer must correspond to zero reflection at the input port 1. Therefore,

$$0 = S_{11} + \frac{(S_{21})^2 S_{22}^*}{1 - |S_{22}|^2}. \tag{3}$$

Reciprocity and the conservation of power require

$$1 = |S_{11}|^2 + |S_{21}|^2 = |S_{22}|^2 + |S_{21}|^2 \tag{4}$$

so that

$$S_{11} = -S_{22}^* \frac{S_{21}^2}{|S_{21}|^2} = M_L \tag{5}$$

or

$$S_{22} = -M_L^* e^{j2\theta} \tag{6}$$

where θ is the angle of S_{21}; *i.e.*,

$$e^{j\theta} = \frac{S_{21}}{|S_{21}|}. \tag{7}$$

Combining (1), (2), (4), and (6) gives

$$\Gamma = \frac{M - M_L}{1 - M_L^* M} e^{-j2\theta}. \tag{8}$$

The phase factor $e^{-j2\theta}$ is not determined by the measurement of a load at port 2, but may be set equal to unity if the position of the reference plane in the secondary transmission line need not be specified.

The phase factor can be determined if a measurement is made with a short circuit (or other phase standard) connected to the secondary port. Only the phase angle of these measurements is required and phase is very insensitive to moderate losses. Therefore, the short-circuit losses and losses of the connection are not nearly as much a matter of concern as when the short-circuit measurement is involved in the determination of the magnitude of the reflection.

Sliding Loads and Characterized Terminations

The quantity M_L is defined as the measurement in the primary connector system when the secondary connector is terminated in a perfect load.

In current practice, the perfect load is commonly simulated by a sliding load connected to the secondary connector. The reflection coefficient at port 1 describes a circle as the load slides. The center of the circle is usually taken as M_L, although this procedure is an approximation: the true standing-wave ratio of the M_L is the geometric mean of the maximum and minimum standing-wave ratio encountered along the circle, subject to a convention that defines the standing-wave ratio from 0 to ∞ [2].

The sliding load is subject to error from reflection at the connector. Another possible source of error in computer-controlled network analyzer implementation is occasional failure of simple algorithms to fit a satisfactory circle to the sliding load data.

Because of these problems and because of the measurement time required, the sliding load is likely eventually to be replaced by characterized fixed loads. A phase reference such as a short circuit is necessary for proper interpretation of a characterized load, but the short circuit need not have low loss.

Let M_P be the measurement at the primary connector when the secondary connector is terminated with a precision characterized load having a known reflection coefficient Γ_P. Also, let M_S be the measurement on the short circuit with a nominal reflection coefficient of -1. From (8),

$$\Gamma_P = \frac{M_P - M_L}{1 - M_L^* M_P} e^{-j2\theta}$$

$$-1 = \frac{M_S - M_L}{1 - M_L^* M_S} e^{-j2\theta}.$$

Therefore,

$$\Gamma_P = \frac{(M_P - M_L)(1 - M_L^* M_S)}{(M_L - M_S)(1 - M_L^* M_P)}. \tag{9}$$

For typical data values, this equation can be quickly solved for M_L by iteration in accordance with

$$\text{new } M_L = M_P + \frac{(M_S - M_L)(1 - M_L^* M_P)\Gamma_P}{1 - M_L^* M_S} \tag{10}$$

with an initial value of 0 for M_L.

Computational Simplification

The method has been presented, for the sake of clarity, as a two-step procedure. First, correct measurements are made at the primary connector, then (5) is applied to obtain reflection coefficients with reference to the secondary transmission line.

Both calculations are bilinear transformations and therefore can be combined into a single bilinear transformation. Such combination reduces the number of calculations and permits one to conserve computer memory by replacing the primary connector correction information with the overall correction information. However, with ample memory and a stable system, it is worthwhile to preserve the primary connector calibration data so that tests can be made quickly through a variety of adapters.

Application to Waveguides

For reproducible low-reflection connections, waveguide flanges excel. However, the number of waveguide types is painfully large and, as a practical expedient, many waveguide components are measured through adapters.

The conventional calibration requires two offset short circuits for each waveguide size (a flat shorting plate is not a dependable low-loss reflection standard). The alternative calibration that has been described requires only a matched load. A flat shorting plate can be used if a phase reference is desired, since loss is not critical.

Discussion

The advantages of the simplified procedure are particularly significant in relation to current practice. A large and possibly major fraction of automatic microwave measurements are now being made on devices with SMA connectors, through adapters to 7-mm precision connectors. A single lossy SMA connection during the present calibration procedure can contaminate all subsequent measurements. The simplified procedure avoids this hazard, although measurements on each device of course still depend upon its own connections.

Once the system with adapter has been calibrated to give corrected measurements of reflection coefficient, it can be used in the usual ways for corrected measurements of transmission coefficient.

Equation (5) has another use as an arbitrary way of dealing with redundant calibration data. Suppose the network analyzer is calibrated at first with three short circuits, to improve the plausibility of the results for highly reflecting devices [1]. If then the matched load is connected, one can pretend that an adapter is involved and apply (5). A new calibration results such that reflection magnitudes are appropriately corrected at high and low values.

Acknowledgment

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Computer-Aided Design of Waveguide Multiplexers

A. E. ATIA

***Abstract*—A procedure based on an analysis algorithm and practical rules is described for the design of waveguide multiplexers. Simple rules, which enable the designer to quickly find a near-optimum solution in a small number of iteration steps, are given. An example of a 6-channel communications multiplexer, which utilizes narrow-bandpass elliptic function waveguide filters, is also included.**

Introduction

In a multicarrier communications satellite repeater, an output multiplexer is normally needed to combine the power outputs from the traveling-wave tube amplifiers. Such a multiplexer must have the smallest possible amount of loss consistent with the required flatness in the passbands of each channel and with the selectivity required for the rejection of adjacent channels. The most suitable configuration for this application is a waveguide-manifold-type multiplexer such as that shown in Fig. 1.

Various relatively simple decoupling techniques have been previously described for the design of such multiplexers [1]. However, these techniques were found to be unsuitable for the present application, partly because extra decoupling resonators may be needed, thus increasing the size, weight, and loss of the multiplexer, and partly because the guard bands are not wide enough, although the filters have narrow bandwidths.

This paper describes a method for computer-aided design of the multiplexer. When separately and individually connected to a matched load and driven by a matched source, all filters used have the same low-pass normalized prototype characteristics. Hence, each filter may be separately tuned prior to multiplexer assembly, thus considerably reducing the effort involved in practical alignment of the multiplexer. Harmful interaction between the filters is eliminated by properly spacing them along the waveguide manifold. This approach closely simulates the process which would be followed in practical experimental design of the multiplexer.

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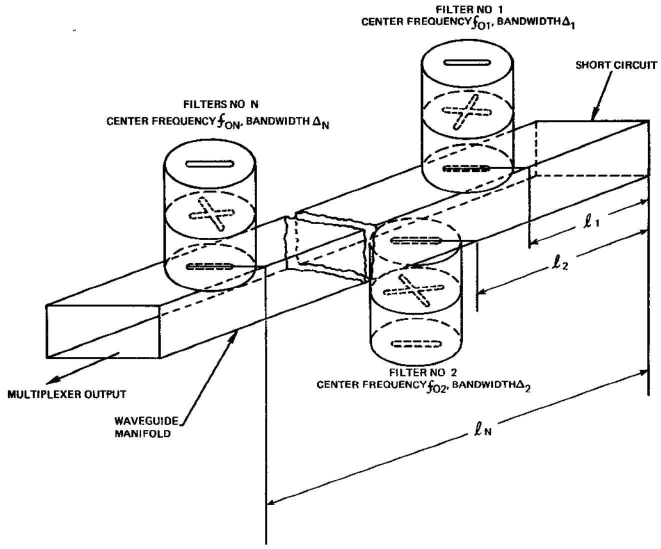

Fig. 1. Waveguide-manifold-type multiplexer.

Multiplexer Configuration and Analysis

The multiplexer to be designed consists of N filters mounted on the wide side of a rectangular waveguide manifold, as shown in Fig. 1. The waveguide is short circuited at one end, while the other end, or output port, is terminated in a matched load. All of the filters are derived from the same normalized prototype equivalent, although they have different center frequencies and bandwidths. The filters are numbered $1,2,\cdots,N$, with filter number 1 nearest to the short circuit and filter N farthest from it. The distance of the centers of the coupling slots of filter number k from the short-circuit end of the waveguide manifold is l_k; its center frequency is f_{0k} and its bandwidth is Δ_k, $k = 1,2,\cdots,N$.

The equivalent circuit of the configuration shown in Fig. 1 can be derived from an equivalent circuit of the filters, such as that shown in Fig. 2 [2], and the equivalent circuit of a T junction of the broad wall of a waveguide. Each of the T junctions may be represented by an E-plane connection [3]. The equivalent circuit parameters B_a and B_b are the same as in [3, p. 365]. Thus the complete equivalent circuit of the multiplexer is as shown in Fig. 3, in which the filters are represented by their lumped element equivalent circuit, the waveguide is represented by dispersive lengths of transmission line, and the junction effects by the susceptances B_a and B_b.

For convenience in the analysis, a set of total voltages and currents and an equivalent set of incident and reflected waves at the junctions of the filter's terminal planes and the waveguide are used in Fig. 3. Furthermore, the analysis is performed for a channel separating multiplexer rather than a summing multiplexer. All impedance levels are normalized to the waveguide characteristic impedance, which is assumed to be unity, and all filters are terminated in equal output loads R_0. Any individual filter can be analyzed to yield its Y parameters when it is considered to be a 2-port network. Only four normalized polynomials, the filter bandwidth, and the center frequency are needed to obtain the Y parameters of any filter [2]. Thus if the kth filter has terminal voltages and currents $[V_1^{(k)},V_2^{(k)}]$ and $[I_1^{(k)},I_2^{(k)}]$, respectively, then the Y matrix of the filter imposes the following constraints:

$$\begin{bmatrix} I_1^{(k)} \\ I_2^{(k)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(k)} & Y_{12}^{(k)} \\ Y_{12}^{(k)} & Y_{22}^{(k)} \end{bmatrix} \begin{bmatrix} V_1^{(k)} \\ V_2^{(k)} \end{bmatrix}$$

and, at the output terminals,

$$V_2^{(k)} = -I_2^{(k)}R_0. \tag{2}$$

Equations (1) and (2) can be solved to yield

$$I_1^{(k)} = U_k V_1^{(k)} \tag{3}$$

where

$$U_k = Y_{11}^{(k)} - \frac{Y_{12}^{(k)2}R_0}{1 + Y_{22}^{(k)}R_0}. \tag{4}$$